

$$= \int \frac{-1}{x} - \frac{1}{x^2} + \frac{1}{x-1} dx$$

$$= -\ln|x| - \frac{x^{-1}}{-1} + \ln|x-1| + C$$

$$= \ln|x-1| - \ln|x| + \frac{1}{x} + C$$

$$\int \frac{5}{4-x^2} dx = \int \frac{A}{2-x} + \frac{B}{2+x} dx \quad (3)$$

بالأسلوب الجزئية

$$A(2+x) + B(2-x) = 5$$

$$x=2 \Rightarrow 4A=5 \Rightarrow A=\frac{5}{4}$$

$$x=-2 \Rightarrow 4B=5 \Rightarrow B=\frac{5}{4}$$

$$= \int \frac{\frac{5}{4}}{2-x} + \frac{\frac{5}{4}}{2+x} \cdot dx$$

$$= -\frac{5}{4} \ln|2-x| + \frac{5}{4} \ln|2+x| + C$$

$$= \frac{5}{4} (\ln|2+x| - \ln|2-x|) + C$$

$$= \frac{5}{4} \ln \left| \frac{2+x}{2-x} \right| + C$$

b

$$\int \frac{2x+1}{x-x^2} dx \quad (1)$$

بالأسلوب الجزئية

$$= \int \frac{2x+1}{x(1-x)} \cdot dx = \int \frac{A}{x} + \frac{B}{1-x} dx$$

$$\Rightarrow A(1-x) + Bx = 2x+1$$

$$x=0 \Rightarrow A=1$$

$$x=1 \Rightarrow B=3$$

$$= \int \frac{1}{x} + \frac{3}{1-x} dx$$

$$= \ln|x| - 3\ln|1-x| + C$$

$$\int \frac{1}{x^2(x-1)} \cdot dx \quad (2)$$

بالأسلوب الجزئية

$$= \int \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} dx$$

$$A x(x-1) + B(x-1) + Cx^2 = 1$$

$$x=0 \Rightarrow B=-1$$

$$x=1 \Rightarrow C=1$$

$$x=-1 \Rightarrow 2A - 2B + C = 1$$

$$2A + 2 + 1 = 1$$

$$A = -1$$

1

$$\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{\tan^2 x + 5 \tan x + 6} dx \quad (5)$$

$$y = \tan x$$

بالتعويض

$$dx = \frac{dy}{\sec^2 x}$$

$$x = \frac{\pi}{4} \quad y = \tan \frac{\pi}{4} = 1$$

$$x = 0 \quad y = \tan 0 = 0$$

$$= \int_0^1 \frac{\sec^2 x}{y^2 + 5y + 6} \cdot \frac{dy}{\sec^2 x}$$

$$= \int_0^1 \frac{1}{(y+3)(y+2)} dy$$

بالتكامل

$$= \ln \left| \frac{y+2}{y+3} \right| \Big|_0^1$$

$$= \ln \frac{3}{4} - \ln \frac{2}{3}$$

$$= \ln \frac{\frac{3}{4}}{\frac{2}{3}} = \ln \frac{9}{8}$$

$\boxed{d}$

$$\int_0^2 \frac{x^2 + 3x + 7}{x+1} \cdot dx \quad (4)$$

بالقسمة

	x	2
x	x^2	$2x$
1	x	2

الباقي
الباقي

$$= \int_0^2 \left(x+2 + \frac{5}{x+1} \right) \cdot dx$$

$$= \frac{x^2}{2} + 2x + 5 \ln |x+1| \Big|_0^2$$

$$= (2+4+5 \ln 3) - (0+0+5 \ln 1)$$

$$= 6 + 5 \ln 3$$

$\boxed{d}$

$$y=1 \Rightarrow 2A=1$$

$$A=\frac{1}{2}$$

$$y=-1 \Rightarrow -2B=1$$

$$B=-\frac{1}{2}$$

$$= \int \frac{\frac{1}{2}}{y-1} - \frac{\frac{1}{2}}{y+1} dy$$

$$= \frac{1}{2} \ln|y-1| - \frac{1}{2} \ln|y+1| + C$$

$$= \frac{1}{2} (\ln|y-1| - \ln|y+1|) + C$$

$$= \frac{1}{2} \ln \left| \frac{y-1}{y+1} \right| + C$$

$$= \frac{1}{2} \ln \left| \frac{x^3-1}{x^3+1} \right| + C$$

b

$$\int \frac{3x^2}{(x^3-1)(x^3+1)} dx \quad (6)$$

بالعوض

$$y = x^3$$

$$dy = \frac{dy}{3x^2}$$

$$= \int \frac{3x^2}{(y-1)(y+1)} \cdot \frac{dy}{3x^2}$$

$$= \int \frac{1}{(y-1)(y+1)} \cdot dy$$

بالطريقة الجزئية

$$= \int \frac{A}{y-1} + \frac{B}{y+1} \cdot dy$$

$$A(y+1) + B(y-1) = 1$$